

# St. Claret College

Roll No:  
Date:

Autonomous, Bengaluru

UG END SEMESTER EXAMINATION-DECEMBER 2024

BCA - I SEMESTER

CA 1324: DISCRETE STRUCTURES

10

TIME: 3 hours.

MAX. MARKS: 80

This paper contains THREE printed pages and FOUR parts

## Instructions:

1. Verify and ensure that the question paper is completely printed.
2. Any discrepancies or questions about the exam paper must be reported to the COE within 1 hour after the examination.
3. Students must check the course title and course code before answering the questions.

## PART-A

Answer ALL questions. Each answer carries ONE mark.

[10 x 1 = 10]

1. What is an antisymmetric relation?  
a)  $(a,a) \in R$  for all  $a \in A$                       b)  $(a,b) \in R$  implies  $(b,a) \notin R$  unless  $a=b$   
c)  $(a,b) \in R$  and  $(b,c) \in R$  implies  $(a,c) \in R$       d)  $(a,b) \in R$  implies  $(b,a) \in R$
2. How many subsets does a set with 5 elements have  
a) 10                      b) 15                      c) 32                      d) 25
3. A function  $f:A \rightarrow B$  is said to be onto (surjective) if  
a) Each element of A maps to a unique element in B.  
b) Every element of B has a preimage in A.  
c)  $f(x)=f(y)$  implies  $x=y$ .  
d)  $f(x)=0 \forall x \in A$ .
4. What is the truth value of the proposition  $p \wedge \neg p$ ?  
a) True                      b) False  
c) Depends on the value of p                      d) Cannot be determined
5. How many ways can you select 3 objects from a set of 7 distinct objects?  
a) 21                      b) 35                      c) 42                      d) 28
6. What happens to the determinant of a matrix if two rows are swapped?  
a) It becomes zero                      b) It doubles  
c) It remains the same                      d) It changes sign
7. Which of the following matrices is singular?  
a)  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$       b)  $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$                       c)  $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$                       d)  $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$



8. Two graphs are said to be isomorphic if
- They have the same number of vertices and edges, but different adjacency matrices.
  - They have the same number of vertices, edges, and the same adjacency matrix.
  - One graph is planar, and the other is not.
  - One graph has directed edges, and the other has undirected edges.
9. If a graph has  $n$  vertices and each vertex has a degree of  $n-1$ , the graph is
- Bipartite
  - Complete
  - A tree
  - Planar
10. A tree is defined as
- Connected graph with no cycles
  - Graph with cycles, multiple components
  - Complete graph
  - Disconnected graph with cycles

### PART-B

Answer any FIVE questions. Each answer carries TWO marks.

[5 x 2 = 10]

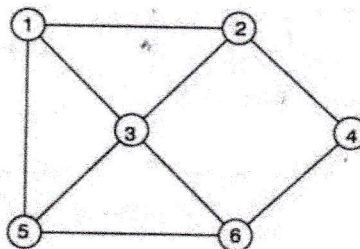
- If  $A = \{a, b\}$  and  $B = \{c, d, e\}$  find  $A \times B$ .
- If  $n(A) = 35$ ,  $n(B) = 24$  and  $n(A \cap B) = 12$  find  $n(A \cup B)$ .
- Construct a truth table for the proposition  $p \vee \sim q$
- Define Contradiction
- Find  $X, Y$  and  $Z$  if  $\begin{bmatrix} 4 - Y & 3 \\ X & 5 \end{bmatrix} = \begin{bmatrix} -6 & Z + 2 \\ 8 & 5 \end{bmatrix}$
- Define the Adjacency Matrix.
- Define pseudo graph with an example?

### PART-C

Answer any FOUR questions. Each answer carries FIVE marks.

[4 x 5 = 20]

- If  $f: R \rightarrow R$  is defined by  $f(x) = 5x + 3$  prove that 'f' is Bijective and also find  $f^{-1}$
- If  $A = \{1, 4\}$ ,  $B = \{2, 3, 6\}$  and  $C = \{2, 3, 7\}$ , verify that  $A \times (B - C) = (A \times B) - (A \times C)$
- Explain Pigeonhole Principle.
- Prove that  $(\sim q \rightarrow \sim p) \leftrightarrow (p \rightarrow q)$  is a tautology
- If  $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$  show that  $A^2 - 4A + 3I = 0$ .
- What is a Hamiltonian circuit? Check whether the following graph contains a Hamiltonian circuit. Justify your answer.



**PART-D**

Answer any FOUR questions. Each answer carries TEN marks.

[4 x 10 = 40]

24. a) In a group of 65 people, 40 like cricket, 10 like both cricket and tennis.  
How many like tennis only and not cricket? How many like tennis?

b) Using Mathematical Induction to prove that

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad (5+5)$$

25. a) Find the number of arrangements of the letters of the word "INDEPENDENCE".  
In how many of these arrangements

(i) Do the words start with P. (ii) Do all the vowels always occur together.

b) If  ${}^{2n}C_3 : {}^nC_2 = 44 : 3$ , find n. (5+5)

26. a) Prove that  $\sim (p \leftrightarrow q) = \sim [(p \rightarrow q) \wedge (q \rightarrow p)]$

b) Construct a Binary search tree 18, 28, 10, 55, 82, 37, 40, 92, 66 (5+5)

27. Find  $A^{-1}$  if  $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

28. a) Find the eigenvalues and the eigenvectors of the matrix  $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

b) Solve by Cramer's rule,  $x - y = 13, x + 3y + 8 = 0$  (5+5)

29. Find the minimum weighted spanning tree by Prim's algorithm.

