Roll No:
Date: __/__/____

St. Claret College

Autonomous, Bengaluru

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UG END SEMESTER EXAMINATION-NOVEMBER 2025
BSC- I SEMESTER
MT 124: MATHEMATICS-I

TIME: 3 hours **MAX. MARKS: 80**

This paper contains FOUR printed pages and FOUR parts

Instructions:
1. **Verify and ensure that the question paper is completely printed.**
2. **Any discrepancies or questions about the exam paper must be reported to the HOD within 1 hour after the examination.**
3. **Students must check the course title and course code before answering the questions.**

PART-A

Answer ALL questions. Each answer carries ONE mark. **[1 x 10 = 10]**

1. Which of the following statements about the eigen values is true?
   a) Eigen values can only be positive
   b) The eigen values of a symmetric matrix are always real.
   c) A matrix can have more eigen values than its order.
   d) Eigen values do not exist for complex matrices.
2. Which among the following matrices is symmetric?
   a) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ b) $\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$
   c) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$
3. In a non-homogeneous system of equations, if $\rho(A) < \rho[A:B]$, what can be concluded?
   a) The system has a unique solution.
   b) The system has infinitely many solutions.
   c) The system has no solution.
   d) The system is consistent.
4. The n^{th} derivative of e^{ax+b}, where a and b are constants is
   a) $a^n e^{ax+b}$
   b) ae^{ax+b}
   c) $(ax+b)e^{ax+b}$
   d) $b^n e^{ax+b}$

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14. Find the n^{th} derivative of $log(1-2x)$.
15. Find the total differential of $f(x,y) = x^3y + x^2y^2 + xy^3$.
16. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $z = e^{\left(\frac{x}{y}\right)}$.
17. Evaluate $\int_0^{\pi/4} (sin^5\theta\, cos^5\theta)\, d\theta$
18. Evaluate $\int_0^{\pi} \frac{sin^4\theta}{(1-cos\theta)^2}\, d\theta$
19. Evaluate $\int_0^{\infty} \frac{dx}{(1+x^2)^4}$
20. Find the equation of the sphere whose centre is $(3,-1,4)$ and which passes through the point $(1,-2,0)$.
21. If the two spheres $x^2 + y^2 + z^2 + 6z - k = 0$ and $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$ cuts orthogonally, find k.
22. If a right circular cone has three mutually perpendicular generators, show that the semi vertical angle is $tan^{-1}\sqrt{2}$.

PART-C

Answer any FOUR questions. Each answer carries FIVE marks. **[5 x 4 = 20]**

23. Show that the equations $x + y + z = 6$, $x + 2y + 3z = 14$, $x + 4y + 7z = 30$ are consistent and solve them.
24. Find the n^{th} derivative of $sin(ax + b)$ where a and b are constants. Also find the n^{th} derivative of $sin(x)$.
25. Verify Euler's theorem for the function $u = \frac{x(x^3-y^3)}{(x^3+y^3)}$.
26. Find the length of the arc of the parabola $y^2 = 4ax$ which is intercepted between the points of intersection with the line $y = 2x$.
27. Show that the length of one loop of the curve $3ay^2 = x(x-a)^2$ is $\frac{4a}{\sqrt{3}}$.
28. Find the equation of the right circular cone generated by revolving the line $\frac{x}{1} = \frac{y}{2} = \frac{2}{3}$ about the line $\frac{x}{-1} = \frac{y}{1} = \frac{z}{2}$.

PART-D

Answer any THREE questions. Each answer carries TEN marks. **[10 x 3 = 30]**

29. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.
30. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$, find A^3 and A^{-2} by using the Cayley Hamilton theorem.
31. a) If $y = a\, cos\left[n\, log\left(\frac{x}{n}\right)\right]$, then by using Leibnitz theorem prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + 2n^2y_n = 0$. (5 marks)

    b) If $x = rcos\theta$ and $y = rsin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$ and $\frac{\partial(r,\theta)}{\partial(x,y)}$. (5 marks)

32. a) Find the surface area of the solid generated when one each of the cycloid $x = a(\theta + sin\theta), y = (1 - cos\theta)$. (5 marks)

b) Find the volume of the solid generated by revolving the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis (5 marks)

33. a) Show that the plane $2x + 3y + 4z = 58$ touches the sphere $x^2 + y^2 + z^2 - 4x - 6y - 8z = 0$ and find the point of contact. (6 marks)

b) Find the equation of a right circular cylinder of radius 2 whose axis passes through (1, 2, 3) and has direction cosines proportional to 2, -3, 6. (4 marks)

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