

Roll No: _____

Date: ____/____/____

St. Claret College

Autonomous, Bengaluru

UG END SEMESTER EXAMINATION-MAY 2025

BSC- II SEMESTER

MT 224: MATHEMATICS- II

TIME: 3 hours

MAX. MARKS: 80

This paper contains THREE printed pages and FOUR parts

Instructions:

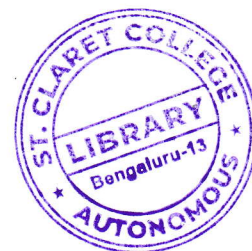
1. Verify and ensure that the question paper is completely printed.
2. Any discrepancies or questions about the exam paper must be reported to the COE within 1 hour after the examination.
3. Students must check the course title and course code before answering the questions.

PART-A

Answer ALL questions. Each answer carries ONE mark.

[1 x 10 = 10]

1. What is the inverse of -1 if $G = \{1, -1, i, -i\}$ is a group under multiplication.
a) -1 b) i c) 1 d) None of the above
2. Which of the following is a cyclic group.
a) $(\mathbb{Z}, +)$ b) $(\mathbb{R}, +)$ c) $(\mathbb{Z}, \cdot)$ d) $(\mathbb{Q}, +)$
3. In a group G , an element 'a' has order 2 if
a) $a^2 = a$ b) $a^2 = e$ c) $a = e$ d) None of the above
4. If φ is angle between radius vector and tangent then
a) $\tan \varphi = r \frac{dr}{d\theta}$ b) $\tan \varphi = \frac{dr}{d\theta}$ c) $\tan \varphi = r \frac{d\theta}{dr}$ d) $\tan \varphi = r_1 \frac{d\theta}{dr}$
5. How many types of asymptotes are there?
a) 3 b) 2 c) 4 d) 1
6. The value of c in Rolle's Theorem where $-\frac{\pi}{2} < c < \frac{\pi}{2}$ and $f(x) = \cos x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is
a) 0 b) π c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$



7. $\lim_{x \rightarrow 3} \frac{x-3}{x^2-2x-3}$ is

- a) 0 b) 1 c) ∞ d) $\frac{1}{4}$

8. A differential equation is considered to be ordinary if it has

- a) One dependent variable
b) More than one dependent variable
c) One independent variable
d) More than one independent variable

9. The integrating factor of the differential equation $\frac{dy}{dx} + xy = x$ is

- a) $e^{-\frac{x^2}{2}}$ b) $e^{\frac{x^2}{2}}$ c) e^x d) None of the above

10. The differential equation $Mdx + Ndy = 0$ will be exact if and only if

- a) $M_y = N_x = 0$ b) $M_y - N_x = 0$ c) $M_x + N_y = 0$ d) $M_x - N_y = 0$

PART-B

Answer any TEN questions. Each answer carries TWO marks.

[2 x 10 = 20]

11. Find the orders of an element of the multiplicative group $\{1, \omega, \omega^2\}$ of cube roots of unity.
12. Find all the generators of Z_{15} .
13. Define right coset and left coset of a group.
14. Show that the angle between radius vector and tangent at any point on the curve $r = ae^{\theta \cot \alpha}$ is constant.
15. For the curve $r = a\theta$ show that $p = \frac{r^2}{\sqrt{r^2 + a^2}}$.
16. Show that the radius of curvature in the pedal form is $\rho = r \frac{dr}{dp}$.
17. Discuss the continuity of $f(x) = x \sin\left(\frac{1}{x}\right)$ at $x=0$.
18. Verify Rolle's Theorem for $f(x) = x^2 - 6x + 8$ in $[2, 4]$.
19. Evaluate $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$.
20. Find the integrating factor for $\frac{dy}{dx} + \frac{2}{x}y = x^3$.
21. Show that the equation $(e^y + 1)\cos x \, dx + e^y \sin x \, dy = 0$ is exact.
22. Solve $\sin px \cos y = \cos px \sin y + p$.

PART C

Answer any FOUR questions. Each answer carries FIVE marks.

[5 x 4 = 20]

23. State and prove Lagrange's theorem.
24. Show that the pair of the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$ intersects orthogonally.
25. Find the pedal equation of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.
26. Discuss the differentiability of the function $f(x) = \begin{cases} x^2, & x \leq 3 \\ 6x - 9, & x > 3 \end{cases}$ at $x = 3$.
27. Expand $f(x) = \tan x$ using Maclaurin's series up to the term containing x^4 .
28. Solve $p^2 + 2p \cot x - y^2 = 0$.

PART D

Answer any THREE questions. Each answer carries TEN marks.

[10 x 3 = 30]

29. a) Prove that the order of every element of a finite group is finite.
b) If G is an abelian group prove that $o(ab) = o(ba) \forall a, b \in G$.
30. Trace the curve cissoids, $y^2(a - x) = x^3$, $a > 0$.
31. State and prove Cauchy's Mean Value theorem. Hence verify the theorem for $f(x) = x^3$ and $g(x) = x^2$ in $[1, 3]$.
32. Solve $p = \tan\left(x - \frac{p}{1+p^2}\right)$.
33. Find the general and singular solution of $(px - y)(x - py) = 2p$ by using the substitution $x^2 = u$ and $y^2 = v$.
