Roll No:
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St. Claret College

Autonomous, Bengaluru

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UG END SEMESTER EXAMINATION-NOVEMBER 2025
BSC- III SEMESTER
MT 325: MATHEMATICS- III

8

TIME: 3 hours **MAX. MARKS: 80**

This paper contains THREE printed pages and FOUR parts

Instructions:
1. **Verify and ensure that the question paper is completely printed.**
2. **Any discrepancies or questions about the exam paper must be reported to the HOD within 1 hour after the examination.**
3. **Students must check the course title and course code before answering the question paper.**

PART-A

Answer ALL questions. Each answer carries ONE mark. **[1 x 10 = 10]**

1. What does a triple integral usually represent?
   a) Area b) Curve length c) Surface area d) Volume in 3D space

2. One of the advantages of changing the order of integration in double integrals is:
   a) It changes the function b) It always increases the result
   c) It may simplify the calculation d) It's mandatory

3. A sequence $\{a_n\}$ converges if:
   a) The terms become unbounded b) The terms go to infinity
   c) The terms approach a fixed real number d) The terms are integers only

4. The sequence $\{a_n\} = \{n\}$ is:
   a) Convergent b) Divergent c) Bounded d) Constant

5. The sequence $\{a_n\} = \left\{\frac{1}{n}\right\}$ converges to:
   a) 1 b) Infinity c) 0 d) Not defined

6. A series $\sum a_n$ converges if:

a) Sequence $\{a_n\}$ is bounded
b) Sequence $\{a_n\}$ is decreasing
c) The partial sums approach a finite limit
d) The terms go to zero rapidly

7. If the terms of a series do not tend to zero, the series:

a) Always converges
b) Might converge
c) Diverges
d) Is constant

8. The geometric series $\sum_{n=0}^{\infty} ar^n$ converges if:

a) $|r| < 1$
b) $|r| > 1$
c) r = 1
d) a = 0

9. The gradient of a scalar function *f(x,y,z)* is a:

a) Scalar
b) Constant
c) Vector
d) Matrix

10. A vector field $\vec{F}$ is conservative if:

a) $\nabla . \vec{F} = 0$
b) $\nabla \times \vec{F} = 0$
c) $\vec{F}$ is constant
d) $\vec{F}$ is zero

PART-B

Answer any TEN questions. Each answer carries TWO marks. **[2 x 10 = 20]**

11. Evaluate $\int_C (5xy\,dx + y^2\,dy)$ where C is the curve in the *xy* plane $y = 2x^2$ from *(0,0)* to *(1,2)*.
12. Evaluate $\int_1^4 \int_0^{\sqrt{4-x}} xy\,dy\,dx$.
13. Define a null sequence.
14. Give an example of a sequence which is oscillating finitely.
15. Find the supremum and infimum of the sequence $\left\{\frac{1}{n}\right\}$.
16. Define a divergent series.
17. Discuss the convergence of the series $\sum_1^{\infty}(-1)^n . n$.
18. State D' Alembert's Ratio Test.
19. Show that the series $1^2 + 2^2 + 3^2 + \cdots + n^2 + \cdots$ diverges to $+\infty$.
20. Gradient of a constant scalar is the _____ vector.
21. If $\varphi(x, y, z) = x^2y^2z^2$ and $\vec{F} = 2x\hat{\imath}+y\hat{\jmath}+3z\hat{k}$, find $\vec{F}.\nabla\varphi$
22. Define vector differential operator.

PART-C

Answer any FOUR questions. Each answer carries FIVE marks. **[5 x 4 = 20]**

23. Evaluate $\int_0^a \int_0^a \int_0^a (x^2 + y^2 + z^2)\,dx\,dy\,dz$.
24. Prove that every convergent sequence has a unique limit.
25. Find the limit of the sequence 0.4, 0.44, 0.444, …

26. Test the convergence of the series $\frac{1}{5}+\frac{\sqrt{2}}{7}+\frac{\sqrt{3}}{9}+\frac{\sqrt{4}}{11}+\cdots$

27. Find the directional derivative of the function $\varphi(x,y,z) = x^2 - 2y^2 + 4z^2$ at (1, 1, -1) in the direction of the vector $2\hat{\imath} + \hat{\jmath} - \hat{k}$.

28. If $\vec{F} = x^2y\hat{\imath} - 2xz\hat{\jmath} + 2yz\hat{k}$. Find curl curl $\vec{F}$.

PART-D

Answer any THREE questions. Each answer carries TEN marks. **[10 x 3 = 30]**

29. a) Evaluate $\int_C (x+y)dx + (y-x)dy$ along the parabola $y^2 = x$ from (1,1) to (4,2). (4 marks)

b) Evaluate $\iint_R \frac{x^2y^2}{x^2+y^2}\,dx\,dy$ where R is the annular region between the circles $x^2 + y^2 = 2$ and $x^2 + y^2 = 1$. (6 marks)

30. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dx\,dy\,dz$. (10 marks)

31. a) Prove that the sequence whose n^{th}term is $\frac{3n+4}{2n+1}$

(i) is monotonically decreasing

(ii) is bounded

(iii) converges to $\frac{3}{2}$ (6 marks)

b) State the Cauchy's General principal of convergence. Show that the sequence whose n^{th}term is $(-1)^n$ is not a Cauchy sequence. (4 marks)

32. a) State and prove comparison test form 1. (5 marks)

b) Test the convergence of the series $\frac{1}{1.2}+\frac{2}{3.4}+\frac{3}{5.6}+\cdots\infty$ (5 marks)

33. a) If $\vec{r}$ is a position vector of the point *P(x, y, z)* and φ is a function of r, where $r = |\vec{r}|$ then $\nabla\varphi(r) = \frac{d\varphi}{dr}\hat{r}$. (5 marks)

b) Show that $\vec{F} = (6xy + z^3)\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational. Find φ such that $\vec{F} = \nabla\varphi$. (5 marks)

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