UG – 171

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VI Semester B.A./B.Sc. Examination, September/October 2022
(Semester Scheme)
(CBCS) (F+R) (2016 – 17 and Onwards)
MATHEMATICS – VII

Time : 3 Hours Max. Marks : 70

***Instruction** : Answer **all** Parts.*

PART – A

Answer **any five** questions. **(5×2=10)**

1. a) In a vector space V over the field F, show that

   $(-C)\alpha = -(C\alpha), \quad \forall\, \alpha \in V, \ \ C \in F$.

   b) Prove that the subset $W = \{ (x_1, x_2, x_3) \,/\, x_1 + x_2 + x_3 = 0 \}$ of the vector space $V_3(R)$ is a subspace of $V_3(R)$.

   c) Show that $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (x + y, x - y)$ is a linear transformation.

   d) Define range space and null space of linear transformation.

   e) Write scalar factors in cylindrical co-ordinate system.

   f) Solve : $\dfrac{dx}{z^2 y} = \dfrac{dy}{z^2 x} = \dfrac{dz}{xy^2}$.

   g) Form a partial differential equation by eliminating the arbitrary constants from $z = (x + a)(y + b)$.

   h) Solve $p^2 + q^2 = 3$.

PART – B

Answer **two full** questions. **(2×10=20)**

2. a) Prove that the intersection of any two subspaces of a vector space V(F) is also a subspace of V. But the union of two subspace of vector space V(F) need not be a subspace of V. Justify.

   b) State and prove the necessary and sufficient condition for a non-empty subset W of a vector space V(F) to be a subspace of V.

OR

P.T.O.

3. a) A set of non-zero vectors $(\alpha_1, \alpha_2, ..., \alpha_n)$ of vector space V(F) is linearly dependent if and only if one of vectors say α_k $(2 \leq k \leq n)$ is expressed as a linear combination of its preceding ones.

   b) Find the dimension and basis of the subspace spanned by the vectors (2, –3, 1), (3, 0, 1), (0, 2, 1), (1, 1, 1) of $V_3(R)$.

4. a) Find the linear transformation $T : V_2(R) \rightarrow V_3(R)$ such that $T(-1, 1) = (-1, 0, 2)$ and $T(2, 1) = (1, 2, 1)$.

   b) Find the linear transformation for the matrix $A = \begin{bmatrix} -1 & 0 \\ 2 & 0 \\ 1 & 3 \end{bmatrix}$ with respect to the bases $B_1 = \{(1, 2, 0), (0, -1, 0), (1, -1, 1)\}$ and $B_2 = \{(1, 0), (2, -1)\}$

OR

5. a) State and prove rank-nullity theorem.

   b) Find the linear transformation $T : R^3 \rightarrow R^3$ defined by $T(e_1) = e_1 - e_2$, $T(e_2) = 2e_1 + e_3$, $T(e_3) = e_1 + e_2 + e_3$. Also find the range space, null space, rank and nullity of T.

PART – C

Answer **two full** questions. **(2×10=20)**

6. a) Verify the condition for integrability and solve
   $3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz = 0.$

   b) Solve $x^2(y - z)p + y^2(z - x)q = z^2 (x - y)$.

OR

7. a) Show that the spherical co-ordinate system is orthogonal curvilinear co-ordinate system.

   b) Express vector $\vec{f} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$ in cylindrical co-ordinates and find f_ρ, f_ϕ and f_z.

8. a) Solve : $\dfrac{dx}{mz - ny} = \dfrac{dy}{nx - lz} = \dfrac{dz}{ly - mx}$.

   b) Solve : $\dfrac{dx}{x(y - z)} = \dfrac{dy}{y(z - x)} = \dfrac{dz}{z(x - y)}$

OR

9. a) Express $\vec{f} = 2x\hat{i} - 2y^2\hat{j} + xz\hat{k}$ in cylindrical co-ordinates system and find f_ρ, f_ϕ, f_z.

   b) Express the vector $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in terms of spherical polar co-ordinates and find f_r, f_θ, f_ϕ.

PART – D

Answer **two full** questions. **(2×10=20)**

10. a) Form the partial differential equation given that $f(x + y + z, x^2 - y^2 - z^2) = 0$.

    b) Solve $x(1 + y)p = y(1 + x)q$.

OR

11. a) Solve $(D^2 + DD' - 6D'^2)z = \cos(2x + y)$.

    b) Solve $p + q = \sin x + \sin y$.

12. a) Solve by Charpit's method $z = pq$.

    b) Solve $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$.

OR

13. a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest to this position, find the displacement $y(x, t)$.

    b) Solve $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$ subjected to the conditions

       i) $u(0, t) = 0$, $u(1, t) = 0$

       ii) $u(x, 0) = x^2 - x$, $0 \le x \le 1$.