CB – 172

78

VI Semester B.A./B.Sc. Examination, August/September 2023
(Fresh) (CBCS) (2022-23 and Onwards)
MATHEMATICS – VIII

Time : 3 Hours | Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

I. Answer **any five** questions : **(5×2=10)**

1) Evaluate $\lim_{z\to 3i} \frac{2z+5}{z^2-2z+3}$.

2) Show that $|z-(2+3i)| \le 5$ represents a circle.

3) Prove that $u = \frac{1}{2}\log(x^2+y^2)$ is harmonic.

4) Define bilinear transformation.

5) State Liouville's theorem.

6) If f(z) is differentiable at $z = z_0$ then prove that f(z) is continuous at $z = z_0$.

7) Find the real root of the equation $x^3 - 2x - 5 = 0$ in (2, 3) in one step by using Regula-Falsi method.

8) Write Newton-Raphson iterative formula.

PART – B

II. Answer **any four** questions : **(4×5=20)**

9) Prove that $|z-1|^2 + |z+1|^2 = 4$ represents a circle and find its centre and radius.

10) Derive the Cauchy-Reimann equations in the form $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial u}{\partial \theta} = r\frac{\partial v}{\partial r}$.

11) Prove that f(z) = log z is analytic and hence $f'(z) = \frac{1}{z}$.

12) Show that $u = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate.

P.T.O.

13) Find the orthogonal trajectories of the family of the curves $e^{-x}\cos y + xy = c$.

14) If $f(z) = u + iv$ is analytic, then prove that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$.

PART – C

III. Answer **any four** questions : **(4×5=20)**

15) Evaluate $\int_C (2y + x^2)dx + (3x - y)dy$ along the curve $x = 2t$ and $y = t^2 + 3$ where $0 \le t \le 1$.

16) State and prove Cauchy's integral theorem.

17) Evaluate $\int_C \frac{e^z dz}{(z-1)(z-2)}$ where C is the curve $|z| = 3$.

18) State and prove fundamental theorem of algebra.

19) Prove that $w = \frac{1}{z}$ transforms a circle to circle or to a straight line.

20) Find the bilinear transformation which maps $z = 0, 1, \infty$ onto $w = 1, -i, -1$.

PART – D

IV. Answer **any four** questions : **(4×5=20)**

21) Using Bisection method to find a real root of the equation $x^3 - 4x + 9 = 0$ correct to three decimal places.

22) Solve $10x + y + z = 12$, $2x + 10y + z = 13$ and $2x + 2y + 10z = 14$ by Gauss Jacobi method.

23) By using Power method, find the largest eigenvalue of the method $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ given $X_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

24) Using Taylor's series method find y at $x = 0.2$ considering the terms upto fourth degree, given $\frac{dy}{dx} = x - y^2$ with $y(0) = 1$.

25) Find the solution of $\frac{dy}{dx} = x + y$ with $y(0) = 1$ for $x = 0.1$ using Euler's modified method.

26) Solve $\frac{dy}{dx} = xy$ given $y(1) = 2$ at $x = 1.2$ by using Runge-Kutta method taking $h = 0.2$.