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CB – 171

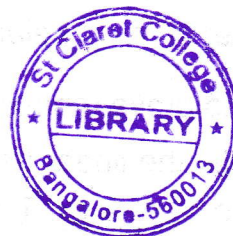
VI Semester B.A./B.Sc. Examination, August/September 2023
(CBCS) (F+R) (2016 – 17 and Onwards)
MATHEMATICS – VII

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all Parts.

PART – A

Answer **any five** questions.

(5×2=10)

1. a) In a Vector Space V over the field F , show that $(-a) \cdot \alpha = -(a \cdot \alpha) \forall a \in F, \alpha \in V$.
- b) Show that $W = \{(0, 0, z) | z \in \mathbb{R}\}$ is a subspace of $V_3(\mathbb{R})$.
- c) Show that the vectors $\alpha_1 = (1, 1, 0)$, $\alpha_2 = (1, 1, 0)$ and $\alpha_3 = (1, 0, 0)$ are linearly independent.
- d) If $T : V_2 \rightarrow V_2$ defined by $T(x, y) = (x + y, y)$, then show that T is a linear transformation.
- e) Write scalar factors in cylindrical co-ordinate system.
- f) Solve $\frac{dx}{zx} = \frac{dy}{yz} = \frac{dz}{xy}$.
- g) Form a partial differential equation by eliminating arbitrary constants from $z = (x - a)^2 + (y - b)^2$.
- h) Solve $p^2 + q^2 = 1$.

PART – B

Answer **two full** questions.

(2×10=20)

2. a) Prove that the intersection of any two subspaces of a vector space $V(F)$ is also a subspace of V . But the union of two subspaces of vector space $V(F)$ need not to be subspace of V . Justify.
- b) State and prove the necessary and sufficient condition for a non-empty subset W of a vector space $V(F)$ to be a subspace of V .

OR

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3. a) Show that the vector $(3, -7, 6)$ is a linear combination of the vectors $(1, -3, 2)$, $(2, 4, 1)$ and $(1, 1, 1)$.
- b) In a n -dimensional vector space $V(F)$, prove that
- any $(n + 1)$ elements of V are linearly dependent.
 - No set of $(n - 1)$ elements can span V .
4. a) State and prove rank-nullity theorem.
- b) Show that the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by $T(e_1) = e_1 + e_2$, $T(e_2) = e_1 - e_2 + e_3$, $T(e_3) = 3e_1 + 4e_3$ is non-singular where $\{e_1, e_2, e_3\}$ is the standard basis of \mathbb{R}^3 .

OR

5. a) Find the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ such that $T(1, 1) = (0, 1, 2)$ and $T(-1, 1) = (2, 1, 0)$.
- b) Find the matrix of linear transformation $T : V_3(\mathbb{R}) \rightarrow V_2(\mathbb{R})$ defined by $T(x, y, z) = (x + y, y + z)$ relative to basis $B_1 = \{(1, 1, 1), (1, 0, 0), (1, 1, 0)\}$ and $B_2 = \{e_1, e_2\}$ of $V_3(\mathbb{R})$ and $V_2(\mathbb{R})$ respectively.

PART – C

Answer **two full** questions.**(2×10=20)**

6. a) Verify the condition of integrability and solve $2yzdx + zxdy - xy(1 + z)dz = 0$.
- b) Solve $(y - z)p + (z - x)q = x - y$.

OR

7. a) Show that cylindrical system is orthogonal curvilinear co-ordinate system.
- b) Express the vector $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in terms of spherical co-ordinates and find f_r, f_θ, f_ϕ .

8. a) Solve $\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}$.

b) Solve $\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$.

OR



9. a) Express $\vec{f} = 2x\hat{i} - 2y^2\hat{j} + xz\hat{k}$ in cylindrical co-ordinates system and find f_ρ, f_ϕ, f_z .
- b) Express $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in the form of spherical polar co-ordinates and find f_r, f_θ, f_ϕ .

PART - D

Answer **two full** questions.**(2×10=20)**

10. a) Form partial differential equation by eliminating arbitrary function from $lx + my + nz = \phi(x^2 + y^2 + z^2)$.
- b) Solve $x(1 + y)p = y(1 + x)q$.

OR

11. a) Solve $(D^2 + DD' - 6(D')^2)z = \cos(2x + y)$
- b) Solve $z^2(p^2z^2 + q^2) = 1$.
12. a) Solve $px + qy = pq$ by Charpit's method.
- b) Solve $(D^2 - DD' - 6(D')^2)z = xy$.

OR

13. a) Solve $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ given that
- i) $u(0, t) = 0, u(l, t) = 0$ for all $t \geq 0$ and
- ii) $u(x, 0) = f(x), \left(\frac{\partial u}{\partial t}\right)_{(x,0)} = \phi(x)$ for $0 < x < l$.
- b) Solve $\frac{\partial u}{\partial t} = 16 \frac{\partial^2 u}{\partial x^2}$ given that
- i) $u(0, t) = 0, u(1, t) = 0$
- ii) $u(x, 0) = x^2 - x, 0 \leq x \leq 1$.
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