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IV Semester B.A./B.Sc. Examination, Sept./Oct. 2022
(Semester Scheme) (CBCS) (2021-22 and Onwards) (F+R)
MATHEMATICS – IV

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all Parts.

PART – A

1. Answer **any five** questions.

(5×2=10)

- a) Define Isomorphism of groups.
- b) Prove that every subgroup of an abelian group is normal.
- c) Define Fourier series.
- d) Verify Rolle's theorem for $f(x) = x^2 - 6x + 8$ in $[2, 4]$.
- e) Evaluate : $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2} \right)$.
- f) Show that $f(x, y) = x^3 + y^3 - 3xy + 1$ is minimum at the point $(1, 1)$.
- g) Solve $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = 0$.
- h) Find the particular integral of $(D^3 + D^2 + D + 1)y = e^{3x+4}$.

PART – B

2. Answer **any two** questions.

(2×5=10)

- a) Prove that a subgroup H of a group G is normal if and only if $ghg^{-1} \in H$, $\forall g \in G$.
- b) Prove that a subgroup H of a group G is normal if and only if every right coset of H in G is a left coset of H in G .
- c) State and prove Fundamental theorem of Homomorphism.

P.T.O.



PART – C

3. Answer **any two** questions.

(2×5=10)

a) Obtain the Fourier series for $f(x) = x^2$ in $-\pi < x < \pi$ and hence deduce

$$\text{that } \frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

b) Find the Fourier series for the function

$$f(x) = \begin{cases} x + \pi/2, & -\pi < x < 0 \\ \pi/2 - x, & 0 \leq x < \pi \end{cases}$$

c) Find the half range sine series for $f(x) = 2x - 1$ over $(0, 2)$.

PART – D

4. Answer **any three** questions.

(3×5=15)

a) Examine the differentiability of

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x \geq 1 \\ 1 - x & \text{for } x < 1 \end{cases} \text{ at } x = 1.$$

b) State and prove Cauchy's mean value theorem.

c) Evaluate :

i) $\lim_{x \rightarrow 0} \log_{\sin x} \sin 2x$

ii) $\lim_{x \rightarrow 0} (\cos x)^{1/x^2}$.

d) Obtain Maclaurin's expansion of the function $e^{\sin x}$.

e) Find the extreme values of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20.$$

PART – E

5. Answer **any three** questions.

(3×5=15)

a) Solve $y'' + 3y' + 2y = \cos^2 x$.

b) Solve $(D^2 - 3D + 2)y = e^x \sin x$.



- c) Solve $x^2 D^2 - 2x(x+1)D + 2(x+1)y = x^3$ given that x is a part of complementary function.
- d) Solve $\cos xy'' + \sin xy' - 2 \cos^2 xy = 2 \cos^5 x$.
- e) Solve $\frac{d^2y}{dx^2} + y = \sec x$ by the method of variation of parameters.

PART – F

6. Answer **any two** questions. (2x5=10)
- a) Sketch the graph of triangular wave and explain it.
 - b) Find the Fourier series representing $f(x) = x$, $0 < x < 2\pi$ and sketch its graphs from $x = -4\pi$ to $x = 4\pi$.
 - c) Sketch the graph of output voltage in Half-wave rectifier and explain it.
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