CS – 166

72

V Semester B.A./B.Sc. Examination, March 2023
(CBCS) (Fresh) (2022-23 and Onwards)
MATHEMATICS – VI(B) (Elective – II)

Time : 3 Hours Max. Marks : 70

Instruction : *Answer* ***all*** *the questions.*

PART – A

Answer **any five** questions. **(5×2=10)**

1. If $a|b$ and $a|c$ then prove that $a|b-c$.

2. Find the G.C.D. of 75 and 243.

3. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then show that $ac \equiv bd \pmod{m}$.

4. Define Euler path.

5. Draw a graph of K_6.

6. What is the maximum number of end vertices a tree on 'n' vertices may have ?

7. Find the complex Fourier transform of $f(x) = \cos(ax+b)^2$. If $F\left[\cos x^2\right] = \frac{1}{\sqrt{2}}\cos\left(\frac{\alpha^2}{4} - \frac{\pi}{4}\right)$.

8. Find the Fourier sine transforms of the function $\frac{\cos x}{\sqrt{x}}$.

PART – B

Answer **any four** questions. **(4×5=20)**

9. Find the G.C.D. of 81 and 237 and express it in a linear combination of 81 and 237.

10. If P is prime and $P|ab$ then show that $P|a$ or $P|b$.

P.T.O.

11. If (ab, c) = 1 then prove that (a,c) = 1.

12. Prove that $3^{500} \equiv 2 \pmod 7$.

13. Find the least non-negative remainder when 9^{10} is divided by 11.

14. Find the least non-negative remainder when $64 \times 65 \times 66$ is divided by 67.

PART – C

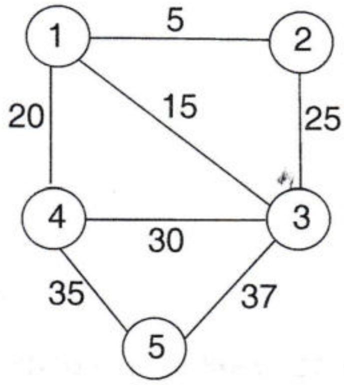
Answer **any four** questions. **(4×5=20)**

15. Define :
    i) Graph
    ii) Self loop
    iii) Pendant vertex
    iv) Isolated vertex
    v) Degree of a vertex

16. Show that a graph G is a tree if and only if G has no cycles and $|E| = |V| - 1$.

17. Prove that every non-trivial tree has atleast 2 vertices of degree 1.

18. Find the minimum weight spanning tree by Prime's algorithm.



19. A tree has five vertices of degree 2, three vertices of degree 3 and four vertices of degree 5. How many vertices of degree 1 does it have ?

20. State and prove hand shaking lemma theorem.

PART – D

Answer **any four** questions. **(4×5=20)**

21. Express $f(x) = \begin{cases} 1 & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$.

22. State and prove convolution theorem.

23. Employing Parseval's identity to the function $f(x) = \begin{cases} 1 & \text{if } |x| < a \\ 0 & \text{if } |x| > a \end{cases}$

    Show that $\int_0^\infty \frac{\sin^2 ax}{x^2} dx = \frac{\pi a}{2}$.

24. Find the function f(x) for which the Fourier sine transform is $\frac{e^{-a\alpha}}{\alpha}$, $a > 0$. Deduce that $F_s^{-1}\left[\frac{1}{x}\right] = \frac{\sqrt{\pi}}{\sqrt{2}}$.

25. Using Parseval's identity for Fourier cosine transform. Show that

    $\int_0^\infty \frac{\sin ax}{x\left(a^2 + x^2\right)} dx = \frac{\pi\left(1 - e^{-a^2}\right)}{2a^2}, a > 0.$

26. Show that $F^{-1}\left[\left(1 - \alpha^2\right) e^{-\left(\alpha^2/2\right)}\right] = x^2 e^{-\left(x^2/2\right)}$.