NP – 225

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II Semester B.Sc. Examination, July/August 2024
(NEP Scheme)
MATHEMATICS
DSC – 2.1 : Algebra – II and Calculus – II

Time : 2½ Hours | Max. Marks : 60

***Instruction** : Answer all Parts.*

PART – A

I. Answer **any four** of the following. **(4×2=8)**

1) Define a subgroup. Give an example.

2) If A = {1, 2, 3} and $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$, $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$, find $(gof)^{-1}$.

3) Verify whether f : (R, +) → (R, +) defined as f(x) = 2x + 1 ∀ x∈R is a homomorphism or not ?

4) If u = xy + yz + zx, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = x + y + 2z$.

5) Show that $f(x, y) = x^3 + y^3 - 3xy + 1$ is minimum at the point (1, 1).

6) Evaluate $\int_0^1 \int_0^2 xy^2 dxdy$.

PART – B

II. Answer **any four** of the following. **(4×5=20)**

7) Prove that order of every element of a finite group is finite.

8) Find all right and left cosets of the subgroup H = {0, 3} in $(Z_6, +_6)$.

9) Prove that a subgroup H of a group G is normal in G if and only if $gHg^{-1} = H \ \forall \ g \in G$.

10) State and prove Euler's theorem for homogeneous function of degree n.

P.T.O.

11) Evaluate $\int_C [(x+y)dx + (y-x)dy]$ along the curve $x = 2t^2 + t + 1$, $y = t^2 + 1$ where $0 \le t \le 1$.

12) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.

PART – C

III. Answer **any four** of the following. **(4×8=32)**

13) Define cyclic group. Prove that every subgroup of a cyclic group is cyclic.

14) Define homomorphism. Verify whether $f : (Z, +) \to (2Z, +)$ defined as $f(x) = 2x \ \forall \ x \in Z$ is an isomorphism. Also find Kernel, if it is a homomorphism.

15) State and prove fundamental theorem of homomorphism.

16) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

17) Expand $f(x, y) = e^x \cos y$ by Maclaurin's series in powers of x and y as far as third degree.

18) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dxdydz}{\sqrt{a^2-x^2-y^2-z^2}}$.